

Two-dimensional electron gas tilt-induced Landau level crossings

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PACS 75.70.Tj – Spin orbit effects

PACS 73.20.At – Electron density of states

PACS 71.70.Di – Landau levels

Abstract – Under a tilted magnetic field \vec{B} , a two-dimensional electron gas (2DEG) with Rashba and Zeeman interactions is considered. The eigenvalues and their degeneracies, derived for various tilt angles, show crossing-free levels except at very high tilt. In this region, the tilt angle becomes the dominant factor in tuning the spin-splitting energy gap rather than the strength of the spin-orbit interaction. Moreover, concomitant with the crossings are the beats found in the 2DEG density of states. The crossings from different levels occur consecutively at around 87° . Similar new observations in Shubnikov-de Haas experimental measurements by Hatke *et al.* [6] attributed such phenomena to an in-plane magnetic field-induced increase in the effective mass. We show here that this behavior is inherent to a 2DEG where spin-orbit interaction and the in-plane magnetic field contribution are taken into account.

Introduction. – The advent of spintronics [1] and the quest to control spin in semiconductors have fueled the current research studies on Landau and spin energy levels of a two-dimensional electron gas (2DEG) [2–6]. Such investigations offer insight on how these energy levels and the spin-orbit coupling of electrons are affected by an applied magnetic field and the 2DEG plane orientation. Tilting of the 2DEG plane relative to the field direction was considered to enhance spin-orbit effects [5,7] making them easier to detect [8].

Early magnetotransport studies in $\text{Ga}_x\text{In}_{1-x}\text{As}/\text{InP}$ [9] at milliKelvin temperatures in strong magnetic fields at a tilt angle θ of the order of $\sim 80^\circ$ showed coincidence of the first two Landau levels (LLs) with low carrier concentration, while the overlap disappears at sufficiently high mobility. This was believed to be caused by electron-electron interactions which shifts the system from a spin-unpolarized state to a spin-polarized state [9]. Other novel effects of tilting has since been observed which are not yet completely understood. For instance, the fractional quantum Hall states in the 2DEG in $\text{GaAs}/\text{AlGaAs}$ heterostructures was found to be suppressed as the sample's tilt angle θ is increased, and resurging only at $\theta = 76^\circ$ with a possible 2DEG nematic phase [3]. Moreover, experiments on the de Haas-van Alphen (dHvA) effect in

high mobility 2DEG yield that magnetization oscillations with respect to an applied magnetic field exhibit beating patterns only at large tilt angles ($\theta \geq 75^\circ$) [10]. The authors attribute these features to spin splitting induced by the structure inversion asymmetric potential in the heterostructure. This type of coupling contribution is known as the Rashba spin-orbit interaction (SOI) [11]. Usually the Rashba SOI is more dominant than the bulk-inversion-asymmetry in III-V [12], II-VI [13] and Si-based [14] semiconductors. This offers a favorable opportunity for practical applications because the Rashba spin-orbit coupling can be controlled via a gate voltage.

In a different study carried out on a 2DEG near the surface of a highly doped p -type InSb crystal, measurements on the differential conductivity (dI/dV) also showed to have beating patterns of the LL intensity for different magnetic fields [2]. Their results on the local density of states were consistent with the Rashba spin splitting. Not only is the Rashba SOI linked with the beats in these Shubnikov-de Haas (SdH) oscillations but it also gives way to a direction-dependent energy spectrum [15]. The authors of this present paper have shown previously that when both Rashba and Zeeman interactions are taken into account there are no beating patterns in the density of states of the 2DEG for $\theta \leq 80^\circ$ [4]. The authors observed that

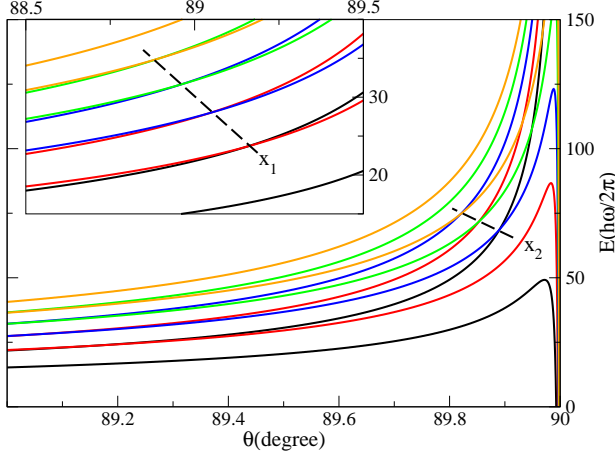


Fig. 1: The eigenvalues as a function of the tilt angle for $n \in [1, 5]$. A pair of identically colored lines indicates the two spin states of a given LL n . The dashed line in the inset points to the first crossing while that at the main frame the second. Here $\alpha = 3.5 \times 10^{-11}$ eVm and $B = 0.1$ T.

depending on the intensity of the applied magnetic field B , the Rashba SOI and Zeeman term signatures can be distinguished [5]. Moreover, at the tilt angles below 80° , the specific heat exhibited a suppressed beating pattern as a function of B .

In this current work, we will distinguish the effects of θ from the rest of the tunable parameters. We will show how the energy spectrum of a two-dimensional electron gas changes as we sweep through the different directions of an applied magnetic field \vec{B} . Special attention is devoted to the large tilt case, that is when $\theta \rightarrow 90^\circ$, as it reveals the beats that seem to have only been restrained in smaller angles [4,5]. Another interesting feature considered is that of a *crossing* which occurs when two adjacent LLs become degenerate for a given applied magnetic field, Rashba SOI strength, and tilt angle. We will show that these crossings are correlated to the beating patterns observed in thermodynamic quantities [10,16].

Recently, SdH oscillations of ultrahigh mobility 2DEG in GaAs/AlGaAs quantum wells showed beating patterns at high tilt angles ($\geq 87^\circ$) which indicate consecutive level crossings occurring at the same angle [6]. They explained their results as due to an increase in the carrier mass induced by the in-plane field with crossings dependent on the filling factor [6]. Here we will show that the density of states of a 2DEG model which incorporates Rashba SOI naturally exhibits multi-crossings at large tilt angles. From this a causal relationship of the LL crossings with the beating patterns observed in thermodynamic quantities is established.

Crossing States. — If Zeeman effect is the lone source of splitting, the SdH or dHvA effect is characterized by a single frequency [17,18]. This is because, just like the Zeeman term, the cyclotron energy $\hbar\omega_c$ is also linearly proportional with the magnetic field magnitude B [17,19],

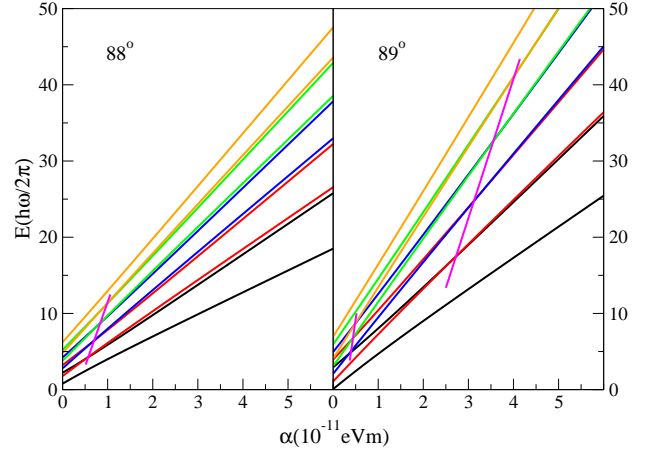


Fig. 2: The eigenvalues as a function of the Rashba SOI for $n \in [1, 5]$. Here $B = 0.1$ T. A pair of identically colored lines indicate the two spin states of a given LL. The magenta lines serve as guides to the eye pointing to the crossing points.

that is, $\omega_c = eB_\perp/m^*$. The perpendicular component B_\perp is set along the z -axis and is equal to $B_z = B \cos(\theta)$ and m^* is the effective mass. When the Rashba SOI is significant, the spin-splitting is not simply linear with B . The Hamiltonian describing a single electron in this system can be written as

$$H = H_0 + H_R + H_Z = \frac{\hbar^2 \vec{k}^2}{2m^*} + \alpha(\vec{\sigma} \times \vec{k}) \cdot \hat{z} - \vec{\mu} \cdot \vec{B}, \quad (1)$$

where H_0 is the free particle energy, H_R is the Rashba spin-orbit interaction, and H_Z is the Zeeman energy. Hence, on top of the strongly degenerate LLs, additional degeneracies can arise if the 2DEG further experiences a spin-orbit interaction. The strength of the Rashba SOI is indicated by the parameter α and is assumed to be constant in this work. Here $\vec{\sigma}$ are the Pauli matrices and the magnitude of the wave vector \vec{k} is determined by $k_j = -i\nabla_j + \frac{e}{\hbar}A_j$, where A_j is the j -th component of the vector potential. The Hamiltonian above is exactly solvable when \vec{B} is normal to the 2DEG. However, if a finite in-plane component, $B_\parallel = B \sin(\theta)$, is present, the problem becomes underspecified and an analytic solution can only be achieved when additional specifications are imposed [4,20].

We circumvent the underspecified problem by imposing equal probability among pairs of LLs that are known to cross when $B_\parallel = 0$ [4,5]. This yields an exact expression for the eigenvalues which is given as

$$E_n^\pm = n + \frac{1}{2} + \frac{\Upsilon}{2} (\sqrt{n+1} + \sqrt{n}) \pm \frac{1}{2} \sqrt{[\Upsilon(\sqrt{n+1} - \sqrt{n}) + 2\Omega_z]^2 + 4\Omega_+ \Omega_-}. \quad (2)$$

Here n is the LL index, $\Omega_j = \mu_B B_j / \hbar\omega_c$ is the j -th component of the rescaled Zeeman energy, $\Upsilon = 2\alpha\sqrt{\zeta}/\hbar\omega_c$ is

the rescaled Rashba parameter, $\zeta = m^*/2\hbar^2$ and $\Omega_{\pm} = \Omega_x \pm i\Omega_y$. A detailed derivation of the eigenvalues can be found in [4]. In the results that follow, energy units are given in terms of the cyclotron energy $\hbar\omega_c$. In addition, the 2DEG is considered to be on the $x-y$ plane subject to an applied field in the $x-z$ direction with magnitude $B = \sqrt{B_x^2 + B_z^2}$. In general, m^* is dependent on the energy due to nonparabolicity, but in this work we let $m^* = 0.05 \times m_e$ (m_e is the free electron mass) which is the effective mass in a 2DEG in InGaAs heterostructures [21] and the same order in other semiconductor systems.

In some theoretical studies [20, 22], the degeneracies at the crossing levels are lifted as θ deviates from zero. That is not the case in this work for large tilt angles as illustrated in fig. 1. Although not included in the figure's scope, a crossing-free spectrum is observed in the region $\theta < \theta_{x_1}$ where θ_{x_1} is the angle where the first crossing x_1 shows up. As θ is further increased more crossings begin to emerge. The crossing point x shifts to lower θ as the LL index n is increased. This is expected since higher n corresponds to a wider spin splitting as indicated in eq. (2).

Inspecting eq. (2), we anticipate an increasing spin gap Δ with increasing θ . This gap between the two spin levels of a given LL ($E_n^+ - E_n^-$) expands until they cross their adjacent LL. The crossings, however, occur only at very large θ as shown in the figure.

It has been predicted that if a finite B_{\parallel} is present, the energy levels do not cross as the Rashba SOI parameter is increased [20]. We check the range of validity of this prediction regarding the energy levels as a function of the Rashba parameter, $E(\alpha)$, using our result in eq. (2). Indeed, for $\theta < \theta_{x_1}$, $E(\alpha)$ is crossing-free [4]. However, for $\theta \geq \theta_{x_1}$, $E(\alpha)$ starts to exhibit crossings. This is depicted in fig. 2. At 88° , only one crossing is resolved because the gap widening is slow to change with respect to the increasing Rashba SOI strength. Relatively larger angles, 89° for example, exhibit a faster spread with α causing the occurrence of multi-crossings. From the plot we understand that the tilt angle has a repulsive effect when $\theta < \theta_{x_1}$ but gives way to crossings when $\theta \geq \theta_{x_1}$.

Comparing fig. 1 and fig. 2, we can extract that large θ and small α both favor crossings. Increasing the Rashba parameter widens both the spin gap and the Landau gap. The interplay of these two produces the crossings at small α and the absence of crossings at large α as depicted in fig. 2. Whether Rashba SOI gives way or inhibits crossings is determined by θ . Note that in both figures, the Zeeman contribution is a constant for a given θ . Otherwise, it is diminished as the tilt increases.

Figure 3 exhibits in detail how the spin gap varies with the Rashba SOI and the field's tilt angle. For the left panel, the $\Delta(\theta)$ curves relative to the tilt angle for two values of α are qualitatively similar in shape. Only the magnitude is increased upon increasing α . However, at very large angles ($\theta > 85^\circ$) the two curves approach the same set of values. This signifies that, although the Rashba SOI strength brings about spin splitting of adjacent levels, the

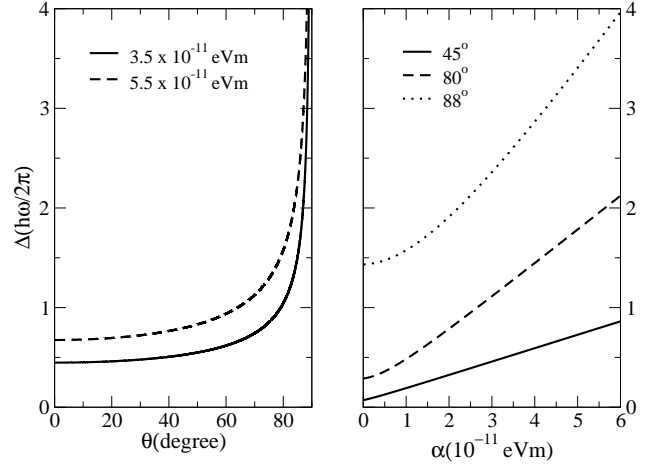


Fig. 3: The spin splitting as a function of the tilt (left plot) and the Rashba parameter (right plot) when $B = 0.1 \text{ T}$.

large field tilting tunes the splitting behavior in addition to α . For the right panel, on the other hand, $\Delta(\alpha)$ is linear with respect to α at small angles. At large angles, however, on top of its increased magnitude, the $\Delta(\alpha)$ shifts from a linear behavior to a polynomial dependence on powers of α . Specifically at $\theta = 88^\circ$, Δ has a cubic dependence on α . Figure 3 affirms how the primary control belongs to θ .

Density of States. — In experiments, the density of states (DOS) are usually probed indirectly through measurements of transport properties such as the magnetization [10] and the specific heat [23]. Recent developments have been able to probe the localized DOS in real space through scanning tunneling spectroscopy [2, 24]. The 2DEG spatially averaged spectrum obtained in Ref. [2] has a beating pattern of the LL intensity in the first subband which the authors attributed to the Rashba spin splitting. Motivated by this observation, including the suppressed beating patterns in the specific heat at small and intermediate angles [5] and the enhanced beats in the magnetization at large tilts [10], the perceptible influence of θ on the DOS in the presence of the Rashba SOI will be considered and shown here. This is done using the phenomenological form of DOS as defined by a series of Gaussian functions, namely,

$$\text{DOS}(E) = \frac{eB_{\perp}}{h} \sum_n \left(\frac{1}{2\pi} \right)^{1/2} \frac{1}{\Gamma} \exp \left[-\frac{(E - E_n^{\pm})^2}{2\Gamma^2} \right], \quad (3)$$

where θ affects the eigenvalues E_n^{\pm} and the magnitude of B_{\perp} .

The broadening width Γ measures how the DOS deviates from its ideal impurity-free 2DEG behavior. Thus, this parameter is indicative of the number of impurities and/or the scale of disorder in the system. Extracted values of the broadening range from 0.4 - 1.5 meV in InAs/GaSb [16], 1.5 - 3.0 meV in InSb/InAlSb [25] and

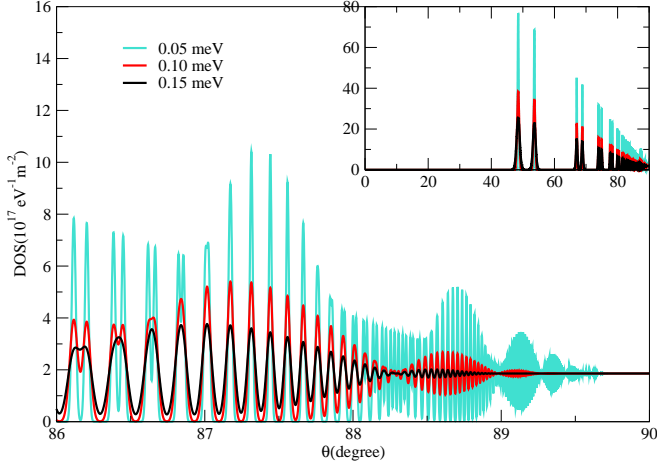


Fig. 4: DOS as a function of the tilt angle at $E \approx 16$ meV for $\alpha = 3.5 \times 10^{-11}$ eVm, $B = 6$ T, and different broadening widths.

can reach as high as ~ 20 meV in InSb [2] depending on the acceptor concentration. The Γ values used here are of the same order as the theoretical fits in AlGaAs/GaAs which are dependent on the field magnitude, $0.1 \text{ meV} \times \sqrt{B(\text{T})}$. For simplicity, we keep Γ constant. The case of $\Gamma < 1.0$ meV ensures that the spin-splitting and the tilt effects are not washed out and dominated by the broadening.

As shown in the inset of fig. 4, the DOS peaks monotonously decreased for $\theta < 86^\circ$. This behavior can be traced to the direct proportionality of the prefactor of eq. (3) to B_\perp . When θ is large as in the case of $\theta > 86^\circ$, the vanishing amplitude at the beat node suggests that the two spin subbands oscillate with similar frequencies. As θ is decreased, the difference between the frequencies increases. When $\theta < 86^\circ$, for example, the two frequencies become very different that only one of them dominates. This gives rise to the single-frequency-DOS. In Ref. [16], the single-frequency oscillation was observed in the high B region of the spectrum while the two-frequency beating pattern was detected in the low B region. This is analogous to our observation where the latter corresponds to the high θ domain. Furthermore, we observe that the DOS oscillations with double frequencies, namely the beats, become more pronounced as B is intensified. This is due to the proportional increase in DOS amplitude.

Referring still to fig. 4, we now relate the beats with the crossings. Both beats and crossings are absent until θ reaches a certain large value ($> 86^\circ$). We deduce that each DOS antinode is concomitant with a crossing point. Since two energy levels merge to the same energy value E , the degeneracy of that E is doubled. This translates to a maximum DOS envelope function. The crossings occur when $\Delta = j\hbar\omega_c$, where $j = 1, 2, 3, \dots$. On the other hand, nodes result when the LLs are equidistant to their adjacent levels. Such a condition is satisfied when $\Delta = (j/2)\hbar\omega_c$,

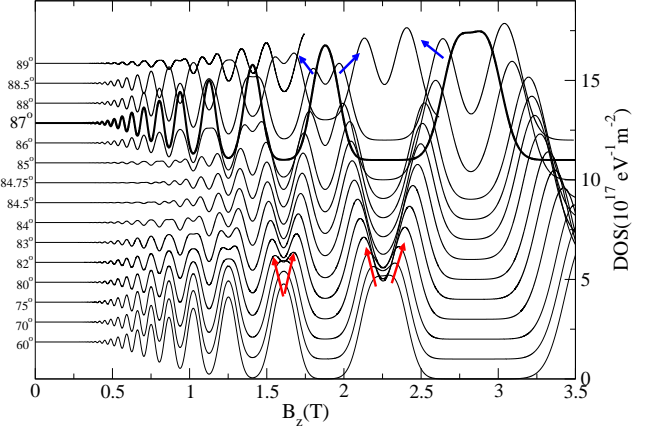


Fig. 5: DOS versus B_\perp at $E \approx 16$ meV showing the phase shift, the spin splitting and the crossings. The curves are shifted for clarity. Here $\alpha = 3.5 \times 10^{-11}$ eVm and $\Gamma = 0.5$ meV. The red arrows point to the spin splitting of a LL while the blue arrows to the reemergence of the spin levels after crossing.

where j here corresponds to the order of the node [17]. The latter resembles the usual evenly spaced harmonic oscillator spectrum.

In fig. 4, we can observe that a large Γ can mask the presence of the beats. This might help explain the absence of SdH beats in materials that are known to have strong Rashba SOI [26]. In addition, when the tilt angle has not yet reached the threshold value θ_{x1} where the beats begin to emerge, a finite B_\parallel only suppresses the beats and the SdH is seen to oscillate with a monotonously decreasing amplitude. In this case, for all $\theta < \theta_{x1}$ the beats are absent.

Collating all of the abovementioned observations, beats are found at the two ends of the tilt spectrum - the zero tilt ($B_\parallel = 0$) [4] and the large tilt ($B_\parallel \rightarrow B$). In between these two extremities, the beats are not observed. Moreover, only at these regions did we also observe the crossings which is indicative of beats and crossings being concomitant. Since the beats are due to the spin-splitting phenomena induced by the Rashba SOI and crossings are found in the case of no tilt [4, 22] but are not found for $\theta < 80^\circ$ [4], then the magnetic field's tilt relative to the 2DEG plane shifts the spin splitting-induced crossings at large tilt as observed in this paper.

To quantify how large the large-tilt region is, we plotted DOS versus $B_\perp = B_z$. See fig. 5. We found out that the DOS versus B_\perp has characteristic 180° phase reversals at $\theta_{r1} \approx 84.75^\circ$, $\theta_{r2} \approx 88.5^\circ$ and $\theta_{r3} \approx 89^\circ$. The DOS are in-phase for $0 < \theta < \theta_{r1}$, for $\theta_{r1} < \theta < \theta_{r2}$, and for $\theta_{r2} < \theta < \theta_{r3}$. The complete phase reversal, with the same frequency, however, is limited only to the small B_\perp region ($B_\perp < 1$ T) where the LL splitting is negligible. This has been observed experimentally [27]. When the splitting becomes evident, it is not straightforward to compare phases

because the LLs begin to split into two spin levels (See red arrows.). As θ is increased, the gap between the spin levels widens until they cross other LLs (See thickened line at $\theta = 87^\circ$). Compared to $\theta = 86^\circ$, each amplitude at $\theta = 87^\circ$ is approximately doubled and the widths of the peaks became narrower. These are evidences of the crossing of two spin levels of neighboring LLs. Note that here we observe multi-level crossings at $\theta = 87^\circ$, specifically at $B_\perp = 1.126$ T, 1.408 T and 1.879 T. This is in agreement with the experimental data of Hatke *et al.*, in which this unusual observation of level crossings occurring at the same angle are attributed to an in-plane magnetic field-induced increase of the carrier mass. Indeed in Ref. [28] the distortion on a Fermi loop was directly measured using the magnetic electron focusing effect in a 2DEG. This, they say, affects the effective mass because of the antisymmetry of the cyclotron orbit. Although, we kept m^* in our work a constant for the simulated data presented herein, the eigenvalues in eq. (2) are derived from first principles. The exact derivation incorporates the Rashba SOI and the in-plane field whose mutual influence on the electron's orbit changes the characteristics of the energy spectrum and consequently, the density of states.

It is important to mention that we do not observe here the anticrossings of adjacent spin-split LLs seen in Ref. [7]. They attribute the anticrossing to the spin-orbit coupling of the collective spin configuration of the quantum hall ferromagnets and to the nonparabolicity causing spin mixing. The authors of [26] believe that it does not come from electron-electron interactions but from the pronounced nonparabolicity of the bands. From our work, we infer that anticrossings must come from nonparabolicity of the energy bands that are induced by other interactions which are not from the structure inversion asymmetric potential and the tilting of the applied field.

In conclusion, we studied how Landau level crossings emerge at large tilting. These crossings bring about beats in the density of states which are manifestations of two spin subbands oscillating at similar frequencies. In agreement with recent experimental data from Hatke *et al.*, the crossings are found to occur consecutively at around 87° . While they attribute this unusual behavior to the in-plane magnetic field-induced carrier mass enhancement, this work has shown that the multiple crossings are intrinsic to a two-dimensional electron gas with Rashba spin-orbit interaction and under intense in-plane magnetic field.

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